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A LANCHESTER MODEL OF SUBMARINE ATTACK
ON A CARRIER BATTLEGROUP

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Abstract

A Lanchester model is developed for a battlegroup ASW engagement. Two variations are included. In the first, long-range missile firing submarines, short-range missile or torpedo firing submarines, and submarines firing only torpedoes distribute their attack uniformly over battlegroup escort ships and carriers. In the second variation, the attack is concentrated on the carriers.

A Lanchester Model of Submarine Attack on a Carrier Battlegroup

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I. Introduction

A Lanchester model is developed for a battlegroup ASW engagement. The model was developed for the Center for Naval Warfare Studies, Naval War College, Newport, RI.

II. General Description

A hypothetical Blue battlegroup consists of two types of platforms: carriers (Z) and escort ships (A). The battlegroup is being attacked by three types of Red submarines: torpedo shooters (V), torpedo or short-range missile shooters (C), and long-range missile shooters (O). When platforms engage, either or both can be destroyed.

The Red submarines regularly return to a Red Base for reloading and repair. The rate of return is proportional to the rate at which the submarines deplete their weapons. Red submarines returning to the Red Base pass through a barrier patrolled by Blue submarine units (S). Submarine vs. submarine engagements occur at this barrier. Red submarines must again pass through the barrier after leaving the Red Base. (See Figure 1.)

All submarines are also subject to area attrition by opposing ASW forces. That is, in each time period a specified fraction of the remaining submarine units are killed. The Blue submarines and escorts are resupplied at specified rates. All other units are not resupplied.

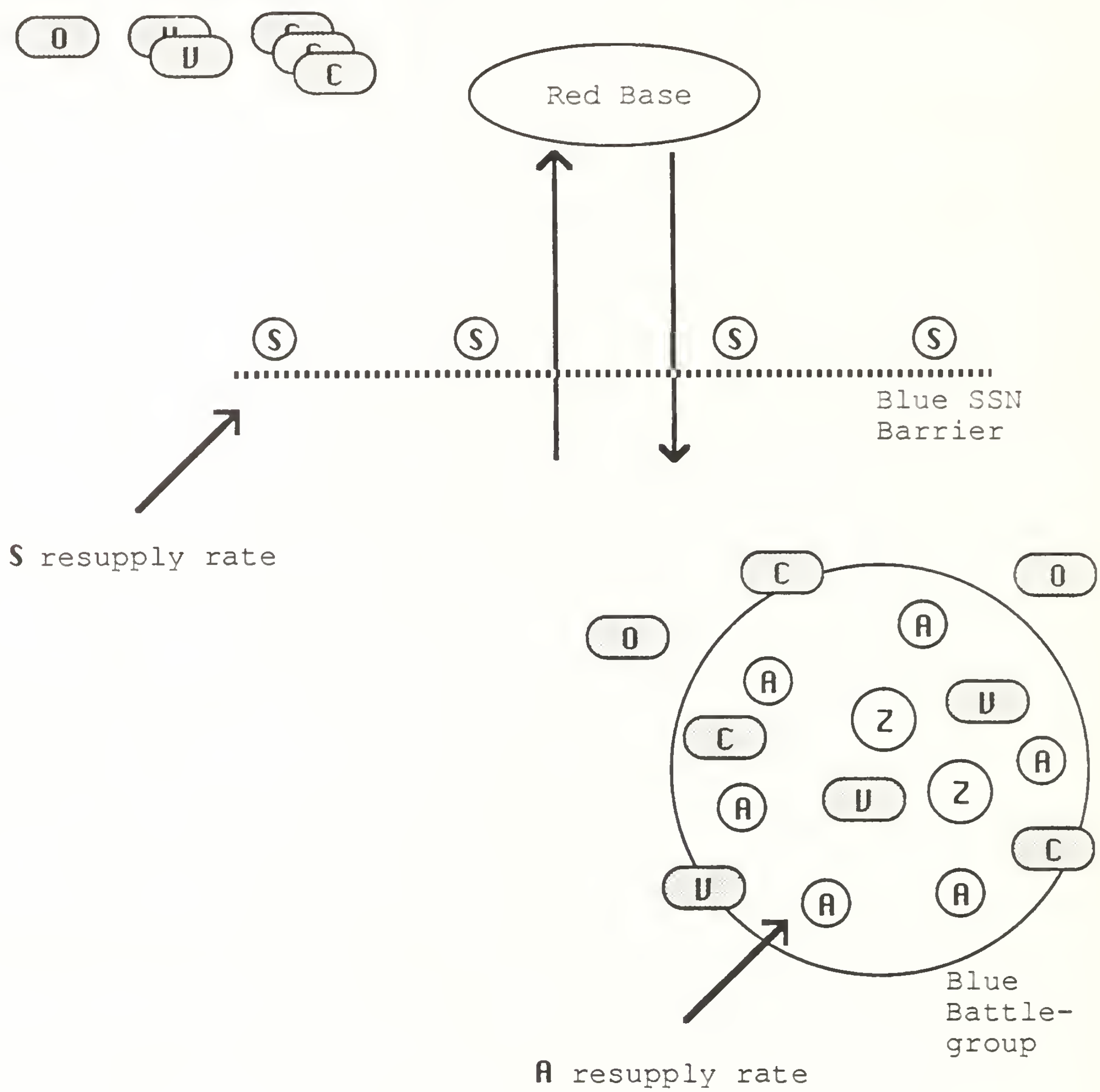

Figure 1. Submarine Attack on a Battlegroup

Table 1. gives the names of the kill probabilities used in the models (kp1-kp21). Table 2. give the remaining variable names.

The initial values for the state variables are A(0), Z(0), S(0), $O_1(0)$, $O_2(0)$, $V_1(0)$, $V_2(0)$, $C_1(0)$, and $C_2(0)$ - all of which must be specified. Two variables are used for each Red submarine. $O_1(t)$, $V_1(t)$, and $C_1(t)$ are the numbers of Red submarine units attacking the battlegroup at time t, while $O_2(t)$, $V_2(t)$, and $C_2(t)$ are the Red submarines at the Red Base.

Platform I	Platform II	Prob{I killed}	Prob {II killed}
Aggressive C w/ torps.	A	kp1	kp2
Evasive C w/ torps.	A	kp3	kp4
C w/ missiles	A	kp5	kp6
Aggressive V w/ torps.	A	kp7	kp8
Evasive V w/ torps.	A	kp9	kp10
O w/ missiles	A	kp11	kp12
Aggressive C w/ torps.	Z	kp13	kp14
C w/ missiles	Z	(same as kp5)	kp16
Aggressive V w/ torps.	Z	kp17	kp18
O w/ missiles	Z	(same as kp11)	kp20
S (in barrier patrol)	O,V,C	kp21	kp22

Table 1. Probabilities of Kill Given an Engagement

R_V: Range of initial detection in a V/A or V/Z engagement (nm).

R_C: Range of initial detection in a C/A or C/Z engagement (nm).

V_{rel}: Average relative speed for random search between Red submarines and Blue surface ships (nm/hr).

area: Area of battlegroup (nm^2).

a_o, a_c: Engagement rates for missile attacks by O and C submarine units (engagements/day).

l_o, l_v, l_c: Fraction of O, V, or C submarine units which deploy from the Red Base each day (1/day).

l_s: Fraction of S submarine units leaving the Blue SSN barrier each day (1/day).

rs_A, rs_S: Resupply rates for A and S units (1/day).

d_v, d_{ct}, d_s: Number of torpedo attacks per patrol that the weapons load allows for V, C, or S units (unitless).

d_o, d_{cm}: Number of missile attacks per patrol that the weapons load allows O or C units (unitless).

deltat: Numerical integration time step (days).

n_o, n_v, n_c, n_s: Fraction of alive O, V, C, or S units killed per day by opposing area ASW forces (1/day).

g_o, g_c: The inverse of the number of A escort ships required to reduce the probability of kill by an O or C missile attack by e^{-1} (1/ship). (A large g means the AAW missile defense is weak; a small g means a strong AAW missile defense.)

$R_{Obar}, R_{Vbar}, R_{Cbar}$: Detection range by an S submarine unit on O, V, or C submarine units at the Blue SSN barrier (nm).

R_{subavg}: The average of R_{Obar}, R_{Vbar}, and R_{Cbar}.

L: Total Blue SSN barrier length (nm).

vel_s, vel_t: Searcher and transitor speeds for the barrier encounter (nm/hr).

Table 2. Variable Names

III. The Lanchester System Models

The models are completely described by expressing the time derivatives of each of the nine state variables as functions of the state variables. Then given initial values for each variable, the values at all subsequent times can be found by numerical integration.

Two Lanchester models are described. The first models a Red submarine attack on all units of the Blue battlegroup. The second models an attack only on the carrier units.

A. Model 1 - Battlegroup Attack

Here the A and Z units of the battlegroup are attacked with missiles by the O_1 and C_1 submarines, and attacked with torpedoes by the V_1 submarines. The defining differential equations follow:

$$\boxed{\begin{aligned}\frac{dA}{dt} = -\frac{2R_v\ V_{rel}}{area} V_1\ A\ kp8 - \frac{A}{A+Z}\left(a_o\ kp12\ e^{-g_o A}\ O_1\right) \\ - \frac{A}{A+Z}\left(a_c\ kp6\ e^{-g_c A}\ C_1\right) + rs_A\end{aligned}}$$

Comments:

1. Escort units (A) are killed by random search interactions with V_1 units plus missile attacks by O_1 and C_1 units.

2. The terms a_o and a_c are engagement (i.e., attack) rates of the two missile-shooting platforms. Constant engagement rates are consistent with a constant cueing rate (from, say, an overhead sensor). Attacks are uniformly distributed over the A and Z units. The O_1 and C_1 platforms are either unable or unwilling to concentrate fire on Z.

3. The two e^{-gA} terms decrease the effectiveness of the missile attacks as the number of defending A units increases.

$$\frac{dZ}{dt} = - \frac{2R_v\ V_{rel}}{area} V_1\ Z\ kp18 - \frac{Z}{A+Z} (a_o\ kp20\ e^{-g_oA}\ O_1) - \frac{Z}{A+Z} (a_c\ kp16\ e^{-g_cA}\ C_1)$$

Comments:

1. Z units are killed in the same manner as A units, but the kill probabilities are different.

2. There is no resupply of carriers.

$$\frac{dS}{dt} = - \min\left\{1, \frac{2R_{subavg}}{L/S} \sqrt{1+\left(\frac{vel_s}{vel_t}\right)^2}\right\} \left(kp21 + (1-kp21)/d_s\right) \left(l_oO_2 + l_vV_2 + l_cC_2 + a_oO_1(1-kp11)/d_o + a_cC_1(1-kp5)/d_{cm} + \frac{2R_v\ V_{rel}}{area} V_1A(1-kp7)/d_v + \frac{2R_v\ V_{rel}}{area} V_1Z(1-kp17)/d_v\right) - n_sS - l_sS + rs_S$$

Comments:

1. $\min\left\{1, \frac{2R_{subavg}}{L/S} \sqrt{1+\left(\frac{vel_s}{vel_t}\right)^2}\right\}$ is the probability that an "average" Red submarine crossing the barrier is detected by an S unit. (Ref: Search and Detection, Alan Washburn, 1981, p.1-11.)

2. $\left(kp21 + (1-kp21)/d_s\right)$ is the probability that an S unit leaves the barrier, given that the S unit detects a transitor. The S unit leaves because it is either killed, or it is not killed but runs out of torpedoes during the attack.

3. $l_oO_2 + l_vV_2 + l_cC_2$ are the rates of O_2, V_2, and C_2 units leaving the Red Base.

4. $a_oO_1(1-kp11)/d_o$ is the rate the O_1 units return to Red Base after running out of missiles. The (1-kp11) term is necessary since a returning O_1 unit must both run out of missiles and not be killed in the counter-attack following the last missile salvo.

5. Similarly, $a_c C_1 (1\text{-}kp5)/d_{cm}$ is the rate C_1 units return to Red Base after running out of missiles.

6. $\frac{2R_v\ V_{rel}}{area}\ V_1 A(1\text{-}kp7)/d_v$ is the returning rate of V_1 units to Red Base after their last interaction with an A unit.

7. $\frac{2R_v\ V_{rel}}{area}\ V_1 Z(1\text{-}kp17)/d_v$ is the returning rate of V_1 units after their last interaction with a Z unit.

8. $n_s S$ is the rate of S kills by Red area ASW forces.

9. $l_s S$ is the rate of S leaving the barrier due to causes other than being killed.

10. rs_S is the resupply rate of S units.

$$\boxed{\begin{gathered}\frac{dO_1}{dt} = l_o q_o O_2 - a_o O_1 \left(kp11 + (1\text{-}kp11)/d_o\right) - n_o O_1 \\ \text{where } q_o = 1 - kp22\ \min\left\{1, \frac{2R_{Obar}}{L/S}\sqrt{1+\left(\frac{vel_s}{vel_t}\right)^2}\right\}\end{gathered}}$$

Comments:

1. q_o is the probability of an O unit surviving a barrier transit against S Blue SSNs. Note that survival could result from no detection by Blue SSNs, or, if detection, an unsuccessful attack.

2. $l_o q_o O_2$ is the rate at which O_1 units arrive at the battlegroup from Red Base.

3. $a_o O_1 \left(kp11 + (1\text{-}kp11)/d_o\right)$ is the rate at which O_1 units leave the battlegroup area due to either being killed after a missile launch or expending all their missiles. (Remember: a_o is the O_1 engagement rate.)

4. $n_o O_1$ is the rate O_1 units are killed by Blue area ASW.

$$\frac{dV_1}{dt} = l_v q_v V_2 - \frac{2R_v\ V_{rel}}{area} V_1 A \ \left(kp7+(1-kp7)/d_v\right) - \frac{2R_v\ V_{rel}}{area} V_1 Z \ \left(kp17+(1-kp17)/d_v\right) - n_v V_1$$

$$\text{where } q_v = 1 - kp22\ \min\left\{1, \frac{2R_{Vbar}}{L/S} \sqrt{1+\left(\frac{vel_s}{vel_t}\right)^2}\right\}$$

Comments:

1. q_v is the probability of a V unit surviving a barrier transit against S Blue SSNs.

2. $\frac{2R_v\ V_{rel}}{area} V_1 A \ \left(kp7+(1-kp7)/d_v\right)$ is the rate V_1 units leave the battlegroup area by being killed in engagements with A units or by running out of torpedoes in such an engagement.

3. Similarly, $\frac{2R_v\ V_{rel}}{area} V_1 Z \ \left(kp17+(1-kp17)/d_v\right)$ is the rate V_1 units leave the battlegroup area due to engagements with Z units.

4. $n_v V_1$ is the rate of area ASW kills of V_1.

$$\frac{dC_1}{dt} = l_c q_c C_2 - a_c C_1 \left(kp5 + (1-kp5)/d_o\right) - n_c C_1$$

$$\text{where } q_c = 1 - kp22\ \min\left\{1, \frac{2R_{Cbar}}{L/S} \sqrt{1+\left(\frac{vel_s}{vel_t}\right)^2}\right\}$$

And at Red Base, we have,

$$\frac{O_2}{dt} = -l_o O_2 + a_o O_1 q_o (1-kp11)/d_o$$

$$\frac{dV_2}{dt} = -l_v V_2 + \frac{2R_v\ V_{rel}}{area} V_1 A q_v (1-kp7)/d_v + \frac{2R_v\ V_{rel}}{area} V_1 Z q_v (1-kp17)/d_v$$

$$\frac{C_2}{dt} = -l_c C_2 + a_c C_1 q_c (1-kp17)/d_{cm}$$

Comment:

1. q_o, q_v, and q_c are defined as before.

2. The rate of change of each of the O_2, V_2, and C_2 groups is the sum of the outgoing rate (e.g., $-l_oO_2$) and the incoming rate due to O_1, V_1, or C_1 running out of weapons and then surviving the Blue barrier.

B. Model 2: Carrier Attack

Now we postulate that the attack is concentrated on the remaining Z units. We require the C_1 platforms shoot only torpedoes. When a torpedo shooter encounters an A unit, it attempts to evade and break contact. Finally, we assume that the O_1 units can direct all of their fire at the Z units. If this is not desired, the model is easily modified to allow any probabilistic split between A and Z.

$$\boxed{\frac{dA}{dt} = -\frac{2R_v\ V_{rel}}{area} V_1A\ kp10 - \frac{2R_c\ V_{rel}}{area} C_1A\ kp4 + rs_A}$$

Comment:

1. The A units now receive no missile fire; and if kp10 and kp4 are small (as would be expected), then A will suffer very little attrition.

$$\boxed{\frac{dZ}{dt} = -\frac{2R_v\ V_{rel}}{area} V_1Z\ kp18 - \frac{2R_c\ V_{rel}}{area} C_1Z\ kp14 - a_o\ kp20\ e^{-g_oA}\ O_1}$$

Comments:

1. The V_1 and C_1 units are conducting random search for Z units.

2. All of O_1's fire is directed at the Z units.

$$\frac{dS}{dt} = -\min\left\{1, \frac{2R_{subavg}}{L/S}\sqrt{1+\left(\frac{vel_s}{vel_t}\right)^2}\right\}\left(kp21 + (1-kp21)/d_s\right)$$

$$\left(l_oO_2 + l_vV_2 + l_cC_2 + a_oO_1(1-kp11)/d_o + \frac{2R_v\ V_{rel}}{area} V_1Z(1-kp17)/d_v + \frac{2R_c\ V_{rel}}{area} C_1Z(1-kp13)/d_{ct}\right)$$

$$- n_sS - l_sS + rs_s$$

Comments:

1. As before, S units are killed in submarine vs. submarine encounters with transitors going both ways through the barrier.

2. Transitors going to the battlegroup do so at rates proportional to their numbers at Red Base.

3. Transitors going to Red Base do so at rates proportional to the rate they run out of weapons.

$$\frac{dO_1}{dt} = l_oq_oO_2 - a_oO_1\left(kp11 + (1-kp11)/d_o\right) - n_oO_1$$

Comment:

1. O_1 units are now aiming at Z units exclusively. But the rate at which O_1 units fire, are killed, and run out of weapons remains as before. So there is no change in the expression for dO_1/dt.

$$\frac{dV_1}{dt} = l_vq_vV_2 - \frac{2R_v\ V_{rel}}{area} V_1A\ kp9 - \frac{2R_v\ V_{rel}}{area} V_1Z\ \left(kp17+(1-kp17)/d_v\right) - n_vV_1$$

Comment:

1. No torpedoes are fired by V_1 units in V_1/A interactions.

$$\frac{dC_1}{dt} = l_cq_cC_2 - \frac{2R_c\ V_{rel}}{area} V_1A\ kp3 - \frac{2R_c\ V_{rel}}{area} C_1Z\ \left(kp13+(1-kp13)/d_{ct}\right) - n_cC_1$$

Comment:

1. Both C_1 and V_1 units are exclusively torpedo shooters.

And at the Red Base,

$$\boxed{\frac{O_2}{dt} = -l_o O_2 + a_o O_1 q_o (1-kp11)/d_o}$$

$$\boxed{\frac{dV_2}{dt} = -l_v V_2 + \frac{2R_v \; V_{rel}}{area} V_1 Z q_v (1-kp17)/d_v}$$

$$\boxed{\frac{C_2}{dt} = -l_c C_2 + \frac{2R_c \; V_{rel}}{area} C_1 Z q_c (1-kp13)/d_{ct}}$$

Comment:

1. As before, the total rate of change of submarine units at Red Base is the arrival rate minus the departure rate.

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